Show that the product of two even signals or of two odd signals is an even signal and that the product of an even and an odd signal is an odd signal.

Let $x(t) = x_1(t)x_2(t)$. If $x_1(t)$ and $x_2(t)$ are both even, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ and $x_2(t)$ are both odd, then

$$x(-t) = x_1(-t)x_2(-t) = -x_1(t)[-x_2(t)] = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ is even and $x_2(t)$ is odd, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)[-x_2(t)] = -x_1(t)x_2(t) = -x(t)$$

and $x(t)$ is odd.

Show that

If $x(t)$ even, then

$$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

We can write

$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$

Letting $t = -\lambda$ in the first integral on the right-hand side, we get

$$\int_{-a}^{0} x(t)\,dt = \int_{a}^{0} x(-\lambda)(-d\lambda) = \int_{0}^{a} x(-\lambda)\,d\lambda$$

Since $x(t)$ is even, that is, $x(-\lambda) = x(\lambda)$, we have

$$\int_{0}^{a} x(-\lambda)\,d\lambda = \int_{0}^{a} x(\lambda)\,d\lambda = \int_{0}^{a} x(t)\,dt$$

Hence,

$$\int_{-a}^{a} x(t)\,dt = \int_{0}^{a} x(t)\,dt + \int_{0}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

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Show that

If $x(t)$ odd, then

$$\int_{-a}^{a} x(t)\,dt = 0$$

$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt = \int_{0}^{a} x(-\lambda)\,d\lambda + \int_{0}^{a} x(t)\,dt$$

$$= -\int_{0}^{a} x(\lambda)\,d\lambda + \int_{0}^{a} x(t)\,dt = -\int_{0}^{a} x(t)\,dt + \int_{0}^{a} x(t)\,dt = 0$$

Show that the complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

is periodic and that its fundamental period is $2\pi/\omega_0$.

$x(t)$ will be periodic if

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t}$$

Since

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

we must have

$$e^{j\omega_0 T} = 1$$

If $\omega_0 = 0$, then $x(t) = 1$, which is periodic for any value of T. If $\omega_0 \neq 0$,

$$\omega_0 T = m2\pi \qquad \text{or} \qquad T = m\frac{2\pi}{\omega_0} \qquad m = \text{positive integer}$$

Thus, the fundamental period T_0, the smallest positive T, of $x(t)$ is given by $2\pi/\omega_0$.

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Show that the sinusoidal signal

$$x(t) = \cos(\omega_0 t + \theta)$$

is periodic and that its fundamental period is $2\pi/\omega_0$.

The sinusoidal signal $x(t)$ will be periodic if

$$\cos[\omega_0(t+T)+\theta] = \cos(\omega_0 t + \theta)$$

We note that

$$\cos[\omega_0(t+T)+\theta] = \cos[\omega_0 t + \theta + \omega_0 T] = \cos(\omega_0 t + \theta)$$

if

$$\omega_0 T = m2\pi \qquad \text{or} \qquad T = m\frac{2\pi}{\omega_0} \qquad m = \text{positive integer}$$

Thus, the fundamental period T_0 of $x(t)$ is given by $2\pi/\omega_0$.

Show that the complex exponential sequence

$$x[n] = e^{j\Omega_0 n}$$

is periodic only if $\Omega_0/2\pi$ is a rational number.

$x[n]$ will be periodic if

$$e^{j\Omega_0(n+N)} = e^{j\Omega_0 n}e^{j\Omega_0 N} = e^{j\Omega_0 n}$$

or

$$e^{j\Omega_0 N} = 1$$

$$\Omega_0 N = m2\pi \qquad m = \text{positive integer}$$

or

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} = \text{rational number}$$

Thus, $x[n]$ is periodic only if $\Omega_0/2\pi$ is a rational number.

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Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. Under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic, and what is the fundamental period of $x(t)$ if it is periodic?

Since $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods T_1 and T_2, respectively, we have

$$x_1(t) = x_1(t + T_1) = x_1(t + mT_1) \qquad m = \text{positive integer}$$
$$x_2(t) = x_2(t + T_2) = x_2(t + kT_2) \qquad k = \text{positive integer}$$

Thus,

$$x(t) = x_1(t + mT_1) + x_2(t + kT_2)$$

In order for $x(t)$ to be periodic with period T, one needs

$$x(t + T) = x_1(t + T) + x_2(t + T) = x_1(t + mT_1) + x_2(t + kT_2)$$

Thus, we must have

$$mT_1 = kT_2 = T$$

or

$$\frac{T_1}{T_2} = \frac{k}{m} = \text{rational number}$$

Let $x_1[n]$ and $x_2[n]$ be periodic sequences with fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] = x_1[n] + x_2[n]$ periodic, and what is the fundamental period of $x[n]$ if it is periodic?

Since $x_1[n]$ and $x_2[n]$ are periodic with fundamental periods N_1 and N_2, respectively, we have

$$x_1[n] = x_1[n + N_1] = x_1[n + mN_1] \qquad m = \text{positive integer}$$
$$x_2[n] = x_2[n + N_2] = x_2[n + kN_2] \qquad k = \text{positive integer}$$

Thus,

$$x[n] = x_1[n + mN_1] + x_2[n + kN_2]$$

In order for $x[n]$ to be periodic with period N, one needs

$$x[n + N] = x_1[n + N] + x_2[n + N] = x_1[n + mN_1] + x_2[n + kN_2]$$

Thus, we must have

$$mN_1 = kN_2 = N$$

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The following equalities are used on many occasions

(a) $$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}$$

(b) $$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$$

(c) $$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \qquad |\alpha| < 1$$

(d) $$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \qquad |\alpha| < 1$$

Continuous Sinusoid	Discrete Sinusoid
① c.t signals are obtd. from transducers & signals sources	① d.t. signals are obtained from c.t signals thro' sampling
② Freq. range of c.t. sinusoid varies from 0 to ∞ expressed in Hz	② d.t. sinusoidal freq. range is limited from $-\pi$ to $+\pi$ expressed in rad/samples, or unitless
③ Both time & freq. are whole numbers or fraction.	③ Period of a d.t sinusoid is always whole numbers
④ c.t. sinusoid freq. = d.t sinusoid freq × sampling freq	④ d.t. sinusoid freq = $\frac{\text{c.t sinusoid}}{\text{sampling freq.}}$
⑤ Ex: $x(t) = A \cos \omega t$	⑤ Ex: $x[nT] = A \cos \Omega nT$

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Even Symmetric	odd Symmetric
① Satisfied $x(-t) = x(t)$ & $x[-n] = x[n]$	① Satisfies $x[-n] = -x[n]$ and $x(-t) = -x(t)$
② $\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$ $\sum_{-K}^{K} x[n] = x[0] + 2\sum_{n=1}^{K} x[n]$	② $\int_{-a}^{a} x(t)\,dt = 0$ $\sum_{-K}^{K} x[n] = 0$
③ If $x_1(t)$ is even, $x_2(t)$ is odd $\int_{-a}^{a} x_1(t)\, x_2(t)\,dt \neq 0$	③ If $x_1(t)$ is even, $x_2(t)$ is odd $\int_{-a}^{a} x_1(t)\, x_2(t)\,dt = 0$
④ Even Symmetric + Constant = Even Symmetric	④ odd Symmetric + Constant = not even, not odd
⑤ $\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} x_e^2(t)\,dt + \int_{-\infty}^{\infty} x_o^2(t)\,dt$	⑤ $\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{-\infty}^{\infty} x_e^2[n] + \sum_{-\infty}^{\infty} x_o^2[n]$
⑥ Even × Even = Even	⑥ odd × odd = Even.

Even and Odd Signals:

A signal $x(t)$ or $x[n]$ is referred to as an *even* signal if

$$x(-t) = x(t)$$
$$x[-n] = x[n]$$

A signal $x(t)$ or $x[n]$ is referred to as an *odd* signal if

$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

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Any signal $x(t)$ or $x[n]$ can be expressed as a sum of two signals, one of which is even and one of which is odd. That is,

$$x(t) = x_e(t) + x_o(t)$$

$$x[n] = x_e[n] + x_o[n]$$

where

$$x_e(t) = \tfrac{1}{2}\{x(t) + x(-t)\} \qquad \text{even part of } x(t)$$

$$x_e[n] = \tfrac{1}{2}\{x[n] + x[-n]\} \qquad \text{even part of } x[n]$$

$$x_o(t) = \tfrac{1}{2}\{x(t) - x(-t)\} \qquad \text{odd part of } x(t)$$

$$x_o[n] = \tfrac{1}{2}\{x[n] - x[-n]\} \qquad \text{odd part of } x[n]$$

Note that the product of two even signals or of two odd signals is an even signal and that the product of an even signal and an odd signal is an odd signal

Signal:- A Signal is formally defined as a function of one or more variables that conveys information on the nature of a physical phenomenon.

System:- A System is formally defined as an entity that manipulates one or more signals to acomplish a function thereby yielding new Signals . . . .

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Energy and Power Signals:

Consider $v(t)$ to be the voltage across a resistor R producing a current $i(t)$. The instantaneous power $p(t)$ per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$

Total energy E and average power P on a per-ohm basis are

$$E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules}$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts}$$

For an arbitrary continuous-time signal $x(t)$, the *normalized energy content* E of $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

The *normalized average power* P of $x(t)$ is defined as

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$

Similarly, for a discrete-time signal $x[n]$, the normalized energy content E of $x[n]$ is defined as

$$E = \sum^{\infty} |x[n]|^2$$

The normalized average power P of $x[n]$ is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x[n]|^2$$

Based on definitions (*1.14*) to (*1.17*), the following classes of signals are defined:

1. $x(t)$ (or $x[n]$) is said to be an *energy* signal (or sequence) if and only if $0 < E < \infty$, and so $P = 0$.
2. $x(t)$ (or $x[n]$) is said to be a *power* signal (or sequence) if and only if $0 < P < \infty$, thus implying that $E = \infty$.
3. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

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Some additional properties of $\delta(t)$ are

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\delta(-t) = \delta(t)$$

$$x(t)\delta(t) = x(0)\delta(t)$$

if $x(t)$ is continuous at $t = 0$.

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

if $x(t)$ is continuous at $t = t_0$.

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)\, dt = x(t_0)$$

$$\delta(t) = u'(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$x[n]\delta[n] = x[0]\delta[n]$$

$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

Reference:

1. Alan V Oppenheim, Alan S, Willsky and A Hamid Nawab, "Signals and Systems" Pearson Education Asia / PHI, 2nd edition, 1997. Indian Reprint 2002.

2. Hwei P Hsu, Schaum's Outline in Theory and Problems of Signals and Systems.

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